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TABLES OF THE HYPERGEOMETRIC DISTRIBUTION FUNCTIONS

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FOREWORD

Tables and graphs based on the hypergeometric distribution are presented for use in determining the confidence interval of the sample estimate of the number of defectives in a finite population. Similarly, the sample size can be determined which would give a certain quality level as the lower bound for a selected confidence level. The hypergeometric distribution is particularly suited for small populations (less than 1,000) where a saving in the sample size is desired even at the expense of some loss in precision of the estimate.

The tables of point and cumulative probabilities are tabulations of selected sample and population combinations. The selected sample sizes range from 4 to 40 and the population, from 50 to 1,000.

For those that have access to an IBM 1401 Model B-4, 8K memory, the computer program is included as Appendix D.

The authors wish to express their appreciation to A. Ohta and K. Thornton for editing and assembly of the tables and to J. Mitchell for supervising the computer tabulation.

CONTENTS

ILLUSTRATIONS

INTRODUCTION

In quality evaluation, the basic question is: "What is the quality level of the stockpile in question?" More often this question is put in the following form: "How large must the sample be to give a certain level of assurance that the stockpile is no worse than X% defective if no defectives are observed in the sample?" In this latter form, the requirement is not for a precise estimate of the stockpile quality level but rather some assurance that the quality is not below a specified level. In this situation, the implication is that there is some willingness to sacrifice some precision if a reduction in the sample size required can be realized.

Most approaches until recently have been based on the binomial distribution. But in cases where the stockpile is small, say less than 500, and the unit item cost high, the binomial has not been a very satisfactory model. As is normally the case in quality evaluation where the populations are small and sampling is without replacement, it appeared that the hypergeometric distribution was the more realistic model to use, but until the advent of the modern-day computer, the formidable task of calculating the probabilities on a desk calculator prevented its use.

Of primary concern to the Oahu Laboratory is coping with small stockpiles of high unit cost weapons. What is desired is a method whereby stockpiles of extremely high or low quality (percent operability) can be readily detected using a minimal size sample. For stockpiles falling in between, additional samples must be tested if greater precision in the quality estimates is desired. As a result, a study was made of a two-stage sampling method based on the hypergeometric distribution and using 95% operability at the 90% confidence level as the lower bound for "good" stockpiles.

APPROACH TO THE PROBLEM

Based on the hypergeometric distribution two main mathematical approaches are proposed.

Approach I

The first approach may be stated in this mathematical form

(1) $$P(D|S,N,M) = \frac{\binom{M}{D}\binom{N-M}{S-D}}{\binom{N}{S}} = \frac{\left[\frac{M!}{(M-D)!\ D!}\right]\left[\frac{(N-M)!}{(N-M-S+D)!\ (S-D)!}\right]}{\frac{N!}{(N-S)!\ S!}}$$

where: S = Sample size
N = Population size
D = Sample defectives
M = Population defectives

and obviously

S -D = Sample Non-defectives
N-M = Population Non-defectives

Equation (1) states that the probability of obtaining D defectives in a sample of size S, given M defectives in a population of N items, is equal to the number of ways of drawing D out of M items times the number of ways of drawing S-D out of N-M items divided by the number of ways of drawing S out of N items. In stockpile quality estimation it is desired to find the confidence interval for the number of defectives in a finite population. Since M is not known, an upper bound on the true M is sought. Call this bound M_u. First assume that $M_u = M_1$. Then look up in an appropriate table (reference (4)) the sum of the probabilities of drawing D or less defectives in the sample. If this sum is less than α, the significance level (e.g. .10), the proper M_u should be less than M_1. Then choose $M_u = M_2$, $M_2 < M_1$ and repeat the above procedure until

$$\sum_{D_i=0}^{D} F(D_i|S,N,M_u-1) \leq \alpha \leq \sum_{D_i=0}^{D} P(D_i|S,N,M_u)$$

Then select M_u-1 or M_u as the upper bound depending on which corresponding sum is closer to α.

The maximum likelihood value is given as $\hat{M} \leq \frac{D}{S}(N+1)$ (reference 1, p. 294) or more completely $\frac{D}{S}(N+1) - 1 \leq \hat{M} \leq \frac{D}{S}(N+1)$ (reference 3, p. 3). References for this approach are found in (1), (2), and (3).

Approach II

The approach followed is to assume that populations with M defectives where M ranges from D to (N-S+D) are tested randomly. In this case $P(D|S,N,M_i)$ is related to $P(D|S,N,M_D)$, $P(D|S,N,M_{D+1})$, ... $P(D|S,N,M_{N-S+D})$. The probability of the observed sample coming from a population with M_i defectives is given as:

$$(2) \qquad P(M_i \mid D,S,N) = \frac{P(D|S,N,M_i)}{\sum_{M=D}^{N-S+D} P(D|S,N,M)} \qquad \text{where, } D \leq M_i \leq N-S+D$$

Equation (2) states that if, from a population of size N, a sample S is drawn and D defectives are observed, the probability of M_i defectives in the population is equal to the ratio of the probability of that set of M_i, N, S, and D to the total sets of N, M_j, S, D where M_j is allowed to range from D to (N-S+D). The number of defectives in the population cannot be less than the number of defectives observed in the sample nor greater than the difference between the total population N, and (S-D) (sample non-defectives).

The denominator $\sum_{M_j=D}^{N-S+D} P(D|S,N,M_j)$ can be shown equal to $\frac{N+1}{S+1}$, a constant. This makes it valid to tabulate P(D S, N, M) instead of P(M S,N,D) for checking purposes. In the form of equation (1), equation (2) becomes the derived equation:

$$(3) \qquad P(M \mid D,S,N) = \frac{\binom{M}{D}\binom{N-M}{S-D}}{\binom{N+1}{S+1}} = \frac{S+1}{N+1} \; P(D|S,N,M)$$

For brevity, let $P(M_i \mid D,S,N) = P(M)$. Then these recurrence relationships are very useful for computational purposes:

$$(4) \qquad \frac{P(M)}{P(M+1)} = \frac{(M-D+1)\ (N-M)}{(M+1)\ (N-M-S+D)}$$

$$(5) \qquad \frac{P(M)}{P(M-1)} = \frac{(M)\ (N-M-S+D+1)}{(M-D)\ (N-M+1)}$$

From (4) and (5) it can be seen that $P(M) > P(M+1)$, where they exist, as long as $(M-D+1)\ (N-M) > (M+1)\ (N-M-S+D)$ and similarly $P(M) > P(M-1)$ as long as $(M)\ (N-M-S+D+1) > (M-D)\ (N-M+1)$. It follows that the maximum likelihood integer $\hat{M}$ may be expressed as:

$$(6) \qquad \frac{D}{S}(N+1) - 1 \leq \hat{M} \leq \frac{D}{S}(N+1)$$

There will be two $\hat{M}$ values where the extreme right and left expressions are: (a) integers and (b) exist. Since equations (4), (5) and (6) show that the probabilities decrease from the maximum likelihood value, serial computations should start with $\hat{M}$ as shown below in equations (7a) and (7b). (Note: $\hat{M}$ differs from that in [Reference 1] and [Reference 3].)

$$(7a) \qquad P(\hat{M}) \geq P(\hat{M}-1) \geq P(\hat{M}-2) \geq \dots\dots P(\hat{M}-k)$$

$$(7b) \qquad P(\hat{M}) \geq P(\hat{M}+1) \geq P(\hat{M}+2) \geq \dots\dots P(\hat{M}+i)$$

$$(7c) \qquad \sum_{R}^{T} P(M_j) = 1 - \alpha$$

If $P(\hat{M}+i) \geq P(\hat{M}-k)$, $P(\hat{M}+i)$ is added to $\sum P(M_j)$ which starts with $P(\hat{M})$; otherwise $P(\hat{M}-k)$ is added. Equation (7c) states that R to T

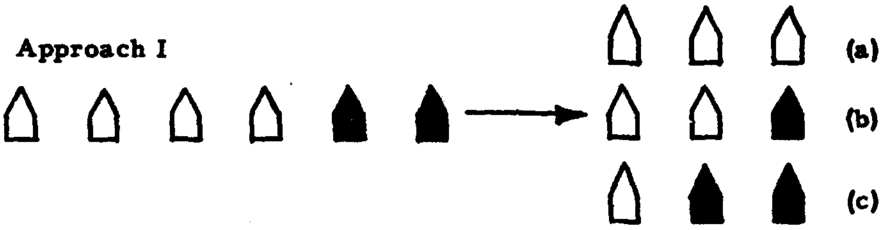
is the range of M defectives in the population when the sum of all the higher probabilities is equal to $1-\alpha$, the confidence level.

$$(8) \qquad \sum_{D}^{L} P(M_j) = 1-\alpha ,$$

L being determined by computations. Equation (8) gives the upper bound of M_j or the so-called one-tail test for small values of D. For $D = 0$ and $D = S$, the one-tail and two-tail tests coincide. For most values of D, the probabilities form an asymmetric as well as a discrete distribution. The asymmetries are illustrated in figure 1 where curved lines are drawn through point probabilities for $N = 50$, $S = 8$, and $D = 0$ to 4.

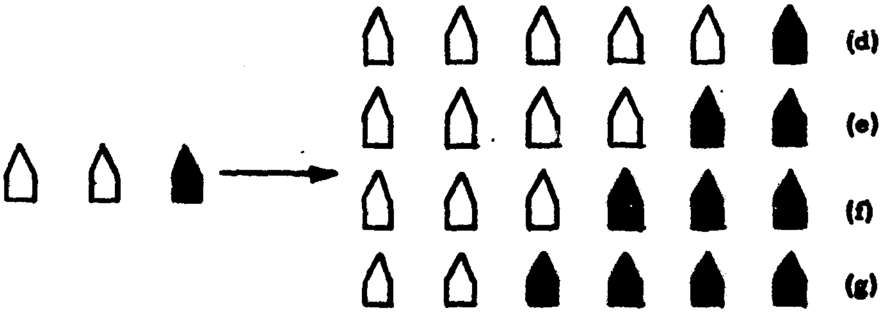
Approaches I and II may perhaps be better illustrated by the use of black and white missiles.

Approach I



There are six missiles in the population - four white and two black. The three possible outcomes in a sample of three are labeled (a), (b), and (c); the chance or probability of drawing any one is determined by equation (1).

Approach II. In stockpile quality evaluation it is desirable to take approach II which is the more realistic statistical model. In this case the population is estimated from a known sample.



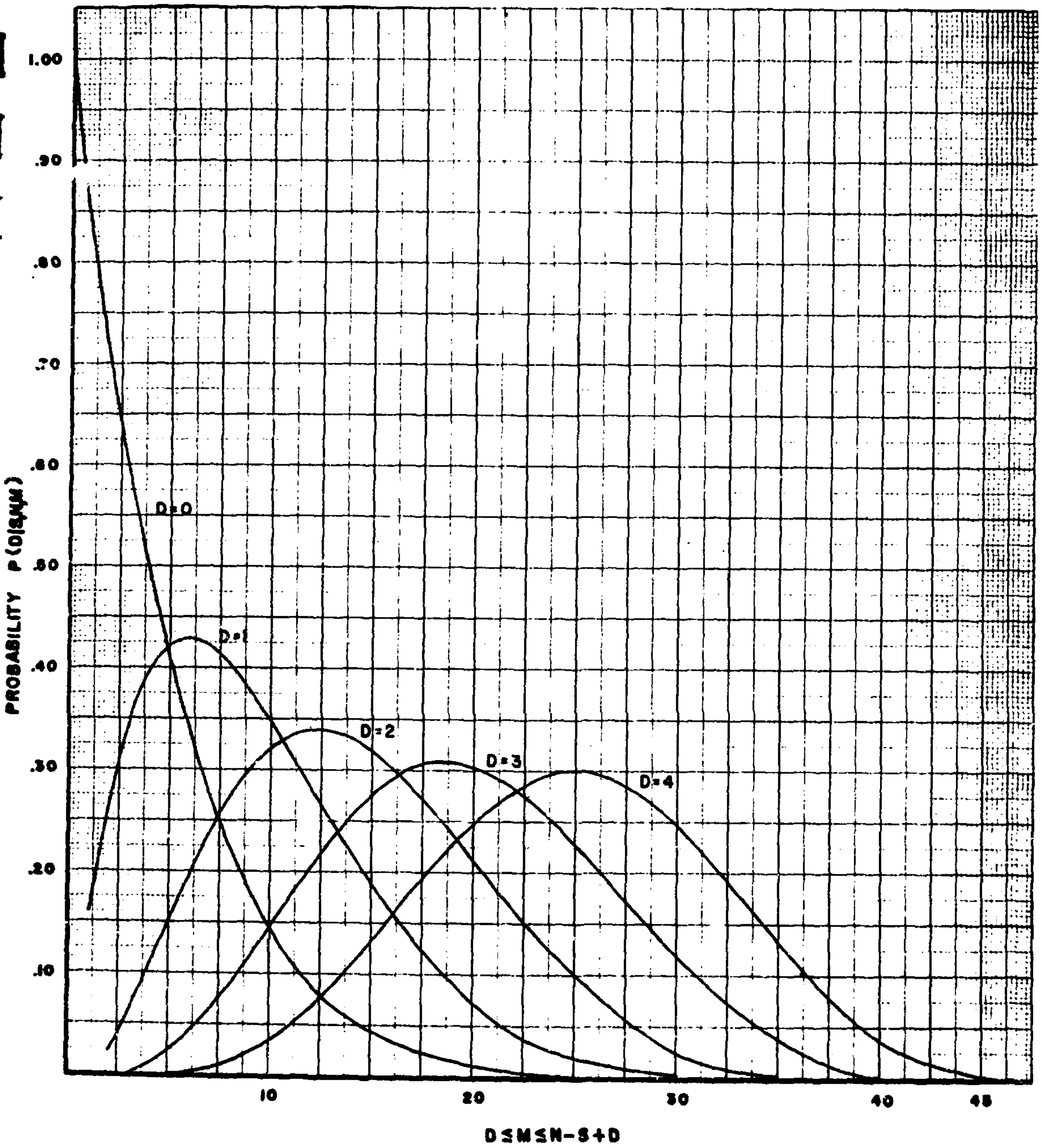


FIGURE 1. CHANGE IN SHAPE OF GENERALIZED POINT PROBABILITY CURVES DUE TO THE NUMBER OF DEFECTIVES PRESENT IN A SAMPLE.

Here, from a population of six, is a sample of three wherein two white and one black missiles were noted. The four possible populations from which this sample could have been drawn are: (d), (e), (f), and (g). The chances or probability of any one of the populations being the one from which this particular sample is drawn is determined by equation (3).

Symmetries

Since $\frac{S+1}{N+1}$ $P(D|S,N,M) = P(M|S,N,D)$ by equation (3), symmetries given by Lieberman and Owen (Reference 4) for $P(D|S,N,M)$ apply in many instances where S and N are fixed.

$$(9) \qquad P(M = M_1 | D_1, S, N) = P(M = N-M_1 | S, S-D_1, N)$$

$$(10) \qquad \sum_{M=A}^{B} P(M|S, D = D_1, N) = \sum_{M=N-A}^{N-B} P(M|S, D = S-D_1, N)$$

Equations (9) and (10) show that tables need only involve half the sample size.

Population defectives M given and sample size S unknown

$$(11) \qquad P(M|D,S,N) = P(S|D,M,N)$$

$$(12) \qquad \sum_{S=D}^{N-M+D} P(S|D,M,N) = \frac{N+1}{M+1}$$

Equations (11) and (12) give the basic equations for the problem of sample size estimation when N, D, and M are known. Any table for M defectives may be used by interchanging S for M.

Two Stage Sampling

Out of a total allowable sample of size S from a population N, an initial sample S_1 is tested and D_1 defectives are found. By means of a

table similar to Appendix C, it is found that x defectives gives the desired percent operability and y defectives ($y>x$) do not. If $D_1 \leq x$ then the lot is accepted. If $D_1 \geq y$, the lot is rejected. If $x<D_1<y$, then the remainder of S, called S_2, is sequentially tested., If at any time a total of D_1+C defectives are found, the lot is rejected, since the total sample S would have at least D_1+C defectives. If the total sample S is tested and (D_1+C-1) defectives are found, the lot is accepted. D_1+C is determined from the probability table for N, S, and M, the number of defectives that will be tolerated.

Best Sample Size S for a Given D

For some values of D sample defectives, a large sample size S is required to reach the desired percent operability for a given confidence level. In these cases, the percent operability should perhaps be lowered to the point where the additional sample units give less than some preselected gain value in percent operability. This is illustrated in figure 2, which is patterned after the graphs in Appendix B.

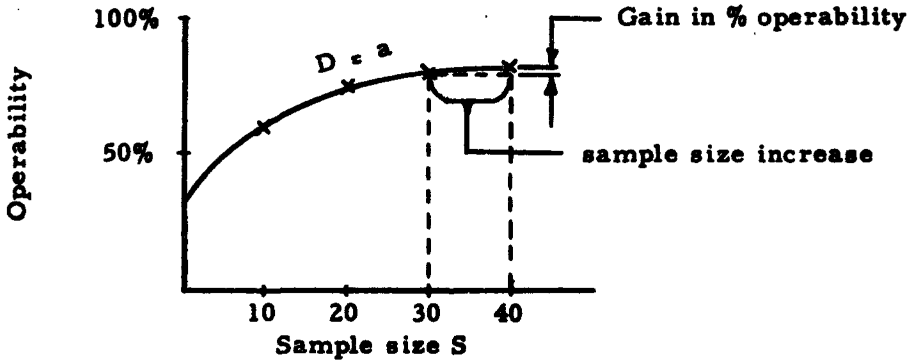


FIGURE 2. SAMPLE SIZE VERSUS OPERABILITY.

Here for D = a, an additional sample of 10, from 30 to 40 will result in a very small gain in percent operability and therefore, S = 30 may be the better choice of sample size.

RESULTS

Tables and Graphs

The table and graphs in Appendices A and B are useful as "quick look" references. The table gives the range for population defectives for certain fixed confidence levels. The graphs give different population lines in terms of percent operability versus fixed sample sizes. Smooth curves are drawn through interpolated points. Only the lower percent operability values are plotted for clarity.

Print-out of Probabilities

A sample of the computer print-out of probabilities is given in Appendix C. Point probabilities and sums for $P(D|S,N,M)$ were tabulated instead of $P(M|D,S,N)$ for purposes of checking with other tables.

The abbreviations are:

S = Sample size
N = Population size
D = Sample defective
M = Population defective
P = $P(D|S,N,M)$

$$\text{SUM} = \sum_{M=R}^{T} P(D|S,N,M)$$

where the sum contains the highest probabilities from R to T

$$\text{CONF} = \frac{\sum_{R}^{T} P(D|S,N,M)}{\sum_{M=D}^{N-S+D} P(D|S,N,M)}$$

$$\text{INTERVAL} = \begin{cases} \text{Lower } M \\ \frac{\text{Lower } M}{N} \\ \text{Upper } M \\ \frac{\text{Upper } M}{N} \end{cases}$$

LEFT SUM = Sum of decreasing probabilities to the left of maximum likelihood

RIGHT SUM = Sum of decreasing probabilities to the right of maximum likelihood

A one-tail test is possible from this type of table. Assume $1-\alpha$ is the confidence level, $Q = \sum_{M=D}^{N-S+D} P(D|S, N, M)$, and

$$R = \sum_{M=\hat{M}+1}^{N-S+D} P(D|S, N, M).$$

Compute αQ and subtract from R. Then trace back in the "Right sum" column until a value just exceeding $R - \alpha Q$ is found. The value of upper M in the same row is the one-tail upper bound on the population defectives.

Program for the Hypergeometric Series

The symbolic language program developed for the IBM 1401 computer, Mod B4, is given in Appendix D. Accuracy in computation of factorials was mainly accomplished by having the decision to multiply by a number from the numerator or divide by a number from the denominator depend upon the number of leading zeros resulting from the previous calculation. Individual probabilities were then usually accurate to ten places and sums of probabilities to eight.

The abbreviations used in the print-out in Appendix D are:

PG LIN = Page and line identification
CT = Count for instruction or reserved storage
OP = Operation instruction
A OPERAND = A or I address of instruction
B OPERAND = B address of instruction
D = D character modification of the basic instruction

REFERENCES

[1] Deming, William Edwards, Some Theory of Sampling, New York, John Wiley and Sons, 1950, p 294

[2] Katz, Leo, Confidence Intervals for the Number Showing a Certain Characteristic in a Population When Sampling is Without Replacement Journal of the American Statistical Association v. 48 (1953), 256-261

[3] Odell, Patrick L., Tables and Graphs for Determining an Upper Confidence Bound on the Number of Defectives in a Finite Population (U)

U. S. Naval Nuclear Ordnance Evaluation Unit, Albuquerque, New Mexico, 1960, p. 1

[4] Lieberman, G. J. and Owen, D. B., Tables of the Hypergeometric Probability Distribution, Stanford University Press, 1961.

APPENDIX A: TABLES OF POPULATION DEFECTIVES FOR CERTAIN CONFIDENCE LEVELS.

N	S	D	90% Confidence Levels		95% Confidence Levels	
			M_L	M_U	M_L	M_U
50	8	0	0	10	0	13
		1	1	17	1	20
		2	4	24	4	27
		3	9	31	7	33
		4	14	37	12	38
	10	0	0	8	0	10
		1	1	14	1	17
		2	3	20	3	23
		3	7	26	6	28
		4	11	31	9	32
	12	0	0	7	0	9
		1	1	12	1	14
		2	3	17	2	19
		3	6	22	5	24
		4	9	26	8	28
60	8	0	0	12	0	16
		1	1	21	1	24
		2	5	30	4	33
		3	10	37	9	40
		4	16	44	14	46
	10	0	0	10	0	13
		1	1	17	1	20
		2	4	25	3	27
		3	8	31	7	34
		4	12	37	11	40
	12	0	0	8	0	11
		1	1	15	1	17
		2	3	21	3	24
		3	7	27	6	29
		4	10	32	9	34
70	8	0	0	15	0	18
		1	1	25	1	29
		2	6	35	5	39
		3	12	44	10	47
		4	18	51	16	54
	10	0	0	12	0	15
		1	1	20	1	24
		2	5	29	4	33
		3	9	36	8	40
		4	14	43	12	46
	12	0	0	10	0	13
		1	1	17	1	20
		2	4	25	3	28
		3	7	31	6	34
		4	12	38	10	40
80	8	0	0	17	0	21
		1	2	29	1	33
		2	7	40	5	44
		3	13	50	11	54
		4	21	59	18	62
	10	0	0	14	0	18
		1	1	24	1	28
		2	5	33	4	37
		3	10	42	8	46
		4	16	50	14	53
	12	0	0	12	0	15
		1	1	20	1	24
		2	4	28	3	32
		3	8	36	7	39
		4	13	43	11	46
90	8	0	0	19	0	24
		1	2	33	1	37
		2	7	45	6	50
		3	15	57	12	60
		4	24	67	20	70
	10	0	0	16	0	20
		1	1	27	1	31
		2	6	38	4	42
		3	11	47	9	51
		4	18	56	16	60
	12	0	0	13	0	17
		1	1	23	1	27
		2	5	32	4	36
		3	9	40	8	44
		4	15	49	13	54
100	8	0	0	21	0	27
		1	2	36	1	42
		2	8	50	6	55
		3	17	63	13	67
		4	26	74	23	78
	10	0	0	18	0	22
		1	2	31	1	35
		2	6	42	5	47
		3	13	53	10	57
		4	20	63	17	67
	12	0	0	15	0	19
		1	1	25	1	30
		2	5	36	4	40
		3	10	45	9	50
		4	16	54	14	58
120	8	0	0	26	0	33
		1	2	44	1	50
		2	10	61	7	67
		3	19	76	16	81
		4	31	89	27	94
	10	0	0	21	0	27
		1	2	37	1	42
		2	7	50	6	57
		3	15	64	12	69
		4	24	76	20	80
	12	0	0	18	0	23
		1	1	31	1	36
		2	6	43	5	49
		3	12	55	10	60
		4	19	65	16	70
	15	0	0	15	0	19
		1	1	25	1	30
		2	5	36	4	41
		3	10	46	8	50
		4	15	54	12	59
	17	0	0	13	0	17
		1	1	22	1	27
		2	4	32	3	36
		3	8	40	7	45
		4	13	48	11	52
140	8	0	0	30	0	38
		1	3	52	2	60
		2	11	71	8	78
		3	22	88	19	94
		4	36	104	31	109
	10	0	0	25	0	32
		1	2	43	1	50
		2	8	59	6	66
		3	17	74	14	81
		4	27	86	23	94
	12	0	0	21	0	27
		1	2	37	1	43
		2	7	51	5	57
		3	14	64	11	70
		4	22	76	19	82
	15	0	0	17	0	22
		1	1	30	1	35
		2	6	42	4	47
		3	11	53	9	58
		4	17	63	15	69
	17	0	0	15	0	20
		1	1	26	1	31
		2	5	37	4	42
		3	10	48	8	52
		4	15	56	13	62
160	8	0	0	35	0	44
		1	3	59	2	68
		2	12	81	9	89
		3	26	101	21	109
		4	41	119	36	125
	10	0	0	29	0	37
		1	2	49	1	57
		2	10	68	7	76
		3	20	86	16	95
		4	31	101	27	108
160	12	0	0	25	0	31
		1	2	43	1	49
		2	8	59	6	66
		3	16	74	13	81
		4	25	87	21	94
	15	0	0	20	0	26
		1	2	35	1	40
		2	6	46	5	55
		3	13	61	10	67
		4	20	73	17	79
	17	0	0	18	0	23
		1	1	30	1	36
		2	5	43	4	49
		3	11	56	9	60
		4	17	65	15	71
180	8	0	0	39	0	50
		1	3	66	2	77
		2	14	92	11	101
		3	29	114	23	122
		4	47	134	39	140
	10	0	0	33	0	41
		1	2	56	2	65
		2	11	77	8	86
		3	22	96	18	104
		4	36	114	30	122
	12	0	0	28	0	35
		1	2	47	1	55
		2	9	66	7	74
		3	18	83	15	91
		4	29	99	24	106
	15	0	0	23	0	29
		1	2	39	1	46
		2	7	54	5	62
		3	14	69	11	75
		4	22	82	19	90
	17	0	0	20	0	26
		1	2	35	1	41
		2	6	49	5	55
		3	12	61	10	68
		4	19	73	16	80
	20	0	0	17	0	22
		1	1	30	1	35
		2	5	42	4	48
		3	10	53	8	59
		4	16	63	14	70
200	8	0	0	44	0	56
		1	3	74	2	88
		2	16	103	11	112
		3	32	127	26	136
		4	51	148	43	156
	10	0	0	37	0	46
		1	3	62	2	72
		2	12	86	9	95
		3	24	107	20	116
		4	39	127	33	135
	12	0	0	31	0	40
		1	2	53	1	62
		2	10	74	7	82
		3	20	93	16	101
		4	32	110	26	118
	15	0	0	26	0	33
		1	2	44	1	51
		2	8	61	6	69
		3	15	76	12	84
		4	26	91	21	99
	17	0	0	23	0	29
		1	2	39	1	46
		2	7	55	5	61
		3	14	69	11	76
		4	22	82	18	89
	20	0	0	19	0	25
		1	1	33	1	39
		2	6	47	4	53
		3	11	59	9	66
		4	18	71	15	77

APPENDIX A - CONTINUED:

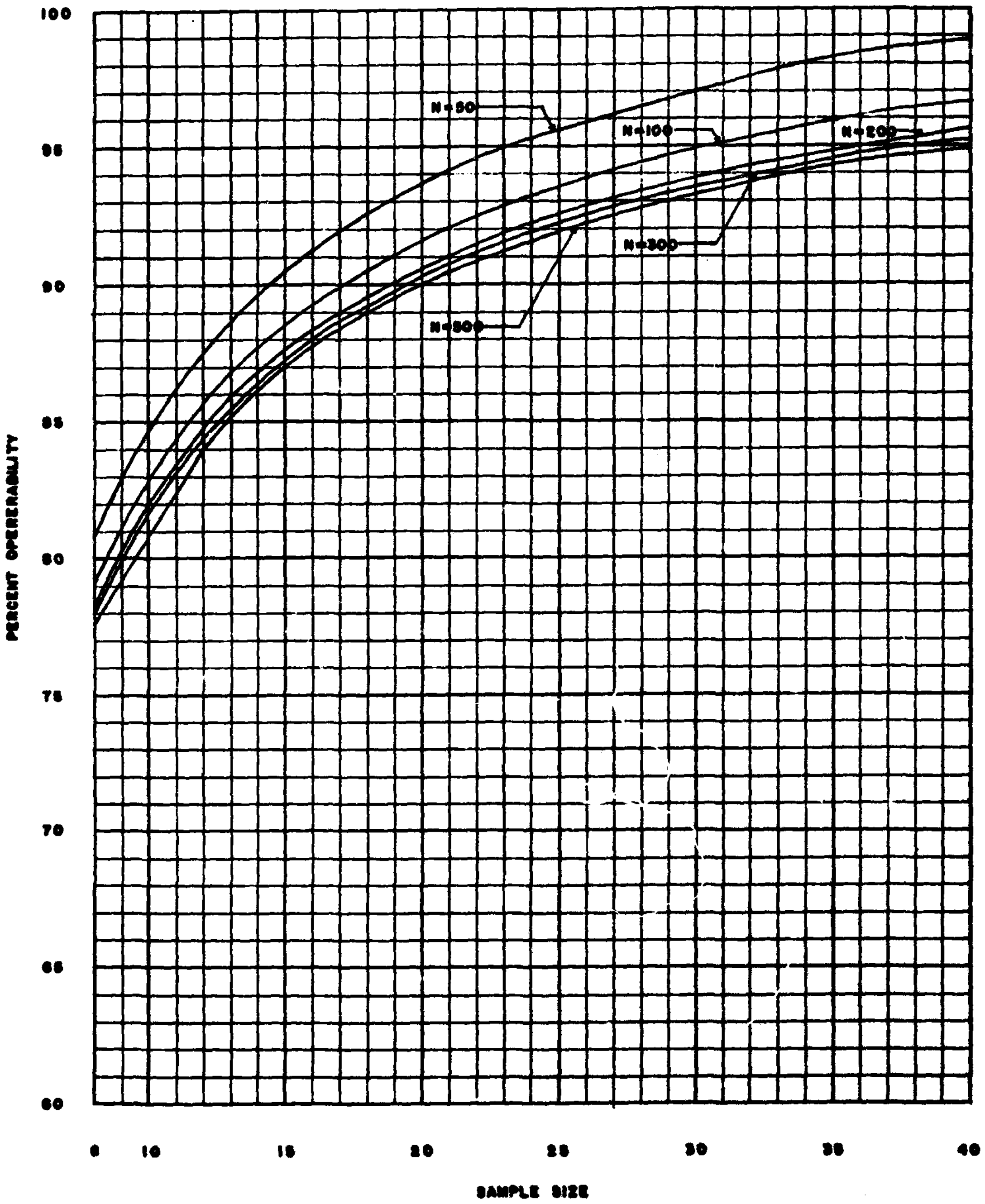
N	S	D	90% Confidence Levels		95% Confidence Levels	
			M_L	M_U	M_L	M_U
225	8	0	0	50	0	62
		1	4	84	2	96
		2	17	114	13	126
		3	34	143	29	153
		4	58	168	49	176
	10	0	0	41	0	52
		1	3	70	2	81
		2	13	96	10	107
		3	27	120	22	131
		4	44	143	37	152
	12	0	0	35	0	45
		1	2	59	2	70
		2	11	83	8	93
		3	22	104	18	114
		4	35	123	30	133
	15	0	0	29	0	37
		1	2	49	1	58
		2	8	68	6	77
		3	17	86	14	95
		4	28	103	23	112
	17	0	0	26	0	33
		1	2	44	1	52
		2	7	61	6	70
		3	15	77	12	86
		4	24	92	20	101
	20	0	0	22	0	28
		1	2	38	1	45
		2	6	52	5	60
		3	13	67	10	74
		4	20	80	17	88
	25	0	0	18	0	23
		1	1	30	1	36
		2	5	43	4	49
		3	10	54	8	60
		4	16	65	14	72
250	8	0	0	55	0	69
		1	4	93	3	107
		2	19	127	14	140
		3	40	159	32	170
		4	64	186	54	196
	10	0	0	46	0	58
		1	3	77	2	90
		2	15	108	11	119
		3	30	134	24	145
		4	49	159	41	169
	12	0	0	39	0	50
		1	3	67	2	76
		2	12	92	9	104
		3	24	116	20	127
		4	39	137	33	148
	15	0	0	32	0	41
		1	2	55	1	64
		2	9	76	7	86
		3	19	96	15	106
		4	30	114	26	124
	17	0	0	29	0	37
		1	2	49	1	58
		2	8	68	6	77
		3	17	86	13	95
		4	27	103	22	112
	20	0	0	25	0	32
		1	2	42	1	50
		2	7	59	5	67
		3	14	74	11	83
		4	22	89	19	98
	25	0	0	20	0	26
		1	1	34	1	40
		2	6	48	4	54
		3	11	60	9	66
		4	18	72	15	80
275	8	0	0	61	0	77
		1	5	103	3	118
		2	21	140	16	155
		3	45	174	35	187
		4	70	205	59	215
275	10	0	0	51	0	64
		1	4	86	2	99
		2	16	118	12	132
		3	33	147	27	160
		4	53	174	45	186
	12	0	0	43	0	55
		1	3	74	2	86
		2	13	102	10	114
		3	27	128	22	140
		4	43	152	36	163
	15	0	0	35	0	45
		1	2	60	1	71
		2	10	84	8	95
		3	21	106	17	117
		4	35	126	30	137
	17	0	0	32	0	40
		1	2	54	1	64
		2	9	75	7	86
		3	18	95	15	105
		4	29	113	24	123
	20	0	0	27	0	35
		1	2	47	1	55
		2	8	65	6	74
		3	15	82	12	91
		4	24	98	20	107
	25	0	0	22	0	28
		1	1	37	1	45
		2	6	53	5	61
		3	12	67	10	75
		4	19	80	16	88
	30	0	0	18	0	24
		1	1	31	1	38
		2	5	44	4	51
		3	10	56	8	63
		4	16	68	14	75
300	8	0	0	67	0	84
		1	5	112	3	129
		2	23	154	17	169
		3	47	190	38	204
		4	77	224	65	236
	10	0	0	55	0	70
		1	4	94	2	108
		2	18	129	13	144
		3	36	161	29	175
		4	58	190	49	203
	12	0	0	47	0	60
		1	3	80	2	94
		2	14	111	10	126
		3	29	139	25	150
		4	47	166	39	178
	15	0	0	39	0	50
		1	2	66	2	78
		2	11	92	8	104
		3	23	116	18	127
		4	36	138	30	148
	17	0	0	35	0	44
		1	2	59	1	69
		2	10	83	7	93
		3	20	104	16	115
		4	32	124	26	135
	20	0	0	30	0	38
		1	2	51	1	60
		2	8	71	6	81
		3	17	90	13	99
		4	26	107	22	116
	25	0	0	24	0	31
		1	2	42	1	49
		2	7	58	5	66
		3	13	73	11	82
		4	21	88	17	96
	30	0	0	20	0	26
		1	1	36	1	42
		2	6	49	4	55
		3	11	61	9	68
		4	17	74	14	82
350	8	0	0	78	0	98
		1	6	131	3	150
		2	26	179	20	198
		3	55	222	45	239
		4	89	261	76	275
	10	0	0	65	0	82
		1	4	109	3	127
		2	20	150	15	168
		3	42	188	34	205
		4	67	222	57	237
	12	0	0	56	0	70
		1	4	94	2	110
		2	17	130	12	146
		3	34	163	27	178
		4	54	193	46	208
	15	0	0	46	0	58
		1	3	78	2	91
		2	13	108	9	121
		3	26	136	21	149
		4	42	161	35	173
	17	0	0	41	0	52
		1	2	69	2	82
		2	11	96	8	109
		3	23	121	18	134
		4	37	144	31	156
	20	0	0	35	0	45
		1	2	60	1	70
		2	9	83	7	95
		3	19	106	15	116
		4	31	126	26	135
	25	0	0	28	0	36
		1	2	49	1	57
		2	8	68	6	78
		3	15	85	12	96
		4	24	102	20	113
	30	0	0	24	0	30
		1	2	41	1	48
		2	6	57	5	64
		3	13	72	10	81
		4	20	87	17	96
	35	0	0	20	0	26
		1	1	35	1	42
		2	5	49	4	56
		3	11	62	9	70
		4	17	75	14	83
400	8	0	0	89	0	112
		1	7	150	4	172
		2	30	206	22	225
		3	63	254	51	273
		4	101	298	86	314
	10	0	0	74	0	96
		1	5	125	3	143
		2	23	172	17	192
		3	48	215	39	233
		4	77	254	64	271
	12	0	0	64	0	81
		1	4	107	2	126
		2	19	149	14	167
		3	39	187	31	204
		4	62	221	52	238
	15	0	0	52	0	67
		1	3	89	2	106
		2	15	123	11	139
		3	30	155	24	171
		4	48	184	40	200
	17	0	0	47	0	60
		1	3	80	2	95
		2	13	111	9	125
		3	26	139	21	154
		4	42	166	35	182
	20	0	0	40	0	51
		1	2	68	1	81
		2	11	96	8	109
		3	22	120	17	133
		4	35	144	29	155

APPENDIX A - CONTINUED:

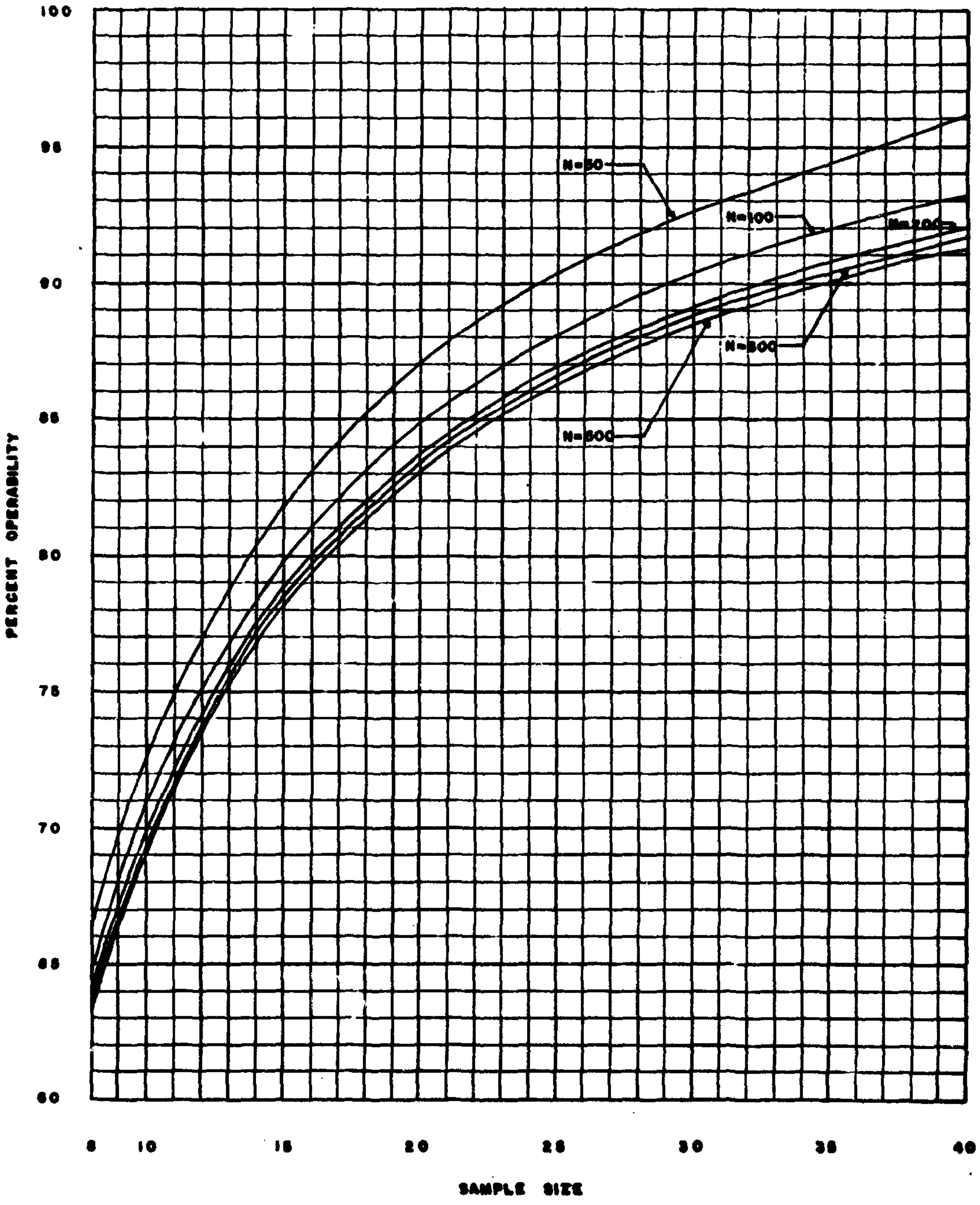
N	S	D	90% Confidence Levels		95% Confidence Levels	
			M_L	M_U	M_L	M_U
400	25	0	0	33	0	42
		1	2	56	1	66
		2	9	70	6	89
		3	17	90	14	110
		4	28	118	23	130
	30	0	0	27	0	35
		1	2	47	1	56
		2	7	65	5	75
		3	15	83	12	93
		4	23	100	19	110
	35	0	0	23	0	30
		1	1	40	1	48
		2	6	56	5	65
		3	12	71	10	80
		4	20	86	16	95
	40	0	0	20	0	26
		1	1	35	1	42
		2	5	49	4	57
		3	11	63	9	71
		4	17	75	14	84
450	5	0	0	100	0	126
		1	7	168	4	193
		2	36	231	25	254
		3	70	286	57	307
		4	114	336	96	353
	10	0	0	84	0	106
		1	6	141	3	164
		2	26	196	19	216
		3	54	243	43	263
		4	86	286	72	306
	12	0	0	72	0	91
		1	4	121	3	142
		2	21	168	15	187
		3	43	209	35	228
		4	70	249	58	268
	15	0	0	59	0	75
		1	4	100	2	118
		2	16	138	12	157
		3	34	175	27	193
		4	54	208	46	226
	17	0	0	53	0	67
		1	3	89	2	106
		2	14	124	10	141
		3	29	156	23	173
		4	47	187	39	204
	20	0	0	45	0	56
		1	3	78	2	92
		2	12	108	9	123
		3	25	136	20	151
		4	39	162	32	177
	25	0	0	37	0	47
		1	3	63	1	74
		2	9	87	7	100
		3	19	110	15	123
		4	31	133	25	146
	30	0	0	31	0	40
		1	2	53	1	63
		2	8	74	6	85
		3	16	93	13	105
		4	26	112	21	126
	35	0	0	26	0	34
		1	2	46	1	56
		2	7	64	5	73
		3	14	81	11	91
		4	22	97	18	107
	40	0	0	23	0	30
		1	1	39	1	48
		2	6	56	4	64
		3	12	71	10	80
		4	19	86	16	95
500	5	0	0	112	0	140
		1	8	187	5	216
		2	39	257	28	283
		3	78	318	63	341
		4	127	374	107	393
500	10	0	0	93	0	118
		1	6	157	4	183
		2	29	216	21	240
		3	60	270	48	293
		4	96	318	80	339
	12	0	0	80	0	101
		1	5	135	3	158
		2	23	186	17	209
		3	48	233	38	255
		4	77	276	64	297
	15	0	0	66	0	84
		1	4	111	2	131
		2	18	154	13	174
		3	37	193	30	214
		4	60	231	50	251
	17	0	0	59	0	75
		1	3	99	2	118
		2	16	139	11	156
		3	32	174	26	193
		4	52	208	43	226
	20	0	0	50	0	65
		1	3	86	2	102
		2	13	119	10	137
		3	27	151	22	168
		4	43	180	36	198
	25	0	0	41	0	53
		1	2	70	1	83
		2	10	97	8	112
		3	21	123	17	138
		4	34	147	28	162
	30	0	0	34	0	46
		1	2	59	1	70
		2	9	82	6	94
		3	18	104	14	117
		4	28	125	23	138
	35	0	0	29	0	38
		1	2	51	1	60
		2	8	71	6	82
		3	15	90	12	101
		4	24	108	20	120
	40	0	0	26	0	33
		1	2	45	1	53
		2	7	63	5	72
		3	13	79	11	90
		4	21	95	17	105
700	5	0	0	157	0	197
		1	11	262	6	302
		2	52	359	39	396
		3	109	446	88	478
		4	177	523	149	550
	10	0	0	131	0	165
		1	8	219	5	256
		2	40	303	29	337
		3	83	378	66	410
		4	134	446	112	476
	12	0	0	112	0	143
		1	7	189	4	222
		2	35	261	25	293
		3	67	327	53	356
		4	108	388	90	418
	15	0	0	92	0	118
		1	5	156	3	186
		2	25	217	18	248
		3	52	272	41	300
		4	83	324	69	352
	17	0	0	83	0	106
		1	5	140	3	166
		2	22	195	16	221
		3	46	246	36	271
		4	72	291	60	316
	20	0	0	71	0	91
		1	4	121	3	143
		2	18	168	13	191
		3	38	212	30	236
		4	60	253	50	276
700	25	0	0	58	0	74
		1	3	99	2	118
		2	14	137	10	157
		3	30	174	23	196
		4	47	207	39	229
	30	0	0	49	0	63
		1	3	84	2	100
		2	12	116	9	133
		3	25	147	19	166
		4	39	176	32	196
	35	0	0	42	0	54
		1	2	71	1	85
		2	10	100	7	118
		3	21	127	17	145
		4	33	152	27	168
	40	0	0	37	0	47
		1	2	63	1	75
		2	9	88	7	102
		3	18	111	15	126
		4	29	134	24	149
1000	5	0	0	225	0	282
		1	16	375	9	432
		2	75	514	56	566
		3	156	638	125	684
		4	253	746	213	787
	10	0	0	188	0	237
		1	12	315	7	366
		2	57	433	41	482
		3	118	540	94	586
		4	191	638	160	681
	12	0	0	161	0	204
		1	10	271	5	317
		2	46	374	33	419
		3	95	468	75	512
		4	153	555	127	597
	15	0	0	133	0	169
		1	7	224	[illegible]	264
		2	35	310	25	350
		3	74	390	58	430
		4	118	463	90	503
	17	0	0	119	0	152
		1	6	201	4	238
		2	31	279	22	316
		3	64	350	50	387
		4	103	417	85	455
	20	0	0	102	0	131
		1	5	173	3	206
		2	26	241	18	274
		3	54	304	42	338
		4	86	363	70	397
	25	0	0	83	0	107
		1	4	142	2	168
		2	20	197	14	228
		3	42	248	33	278
		4	67	297	55	328
	30	0	0	70	0	90
		1	4	120	2	143
		2	17	167	12	191
		3	35	211	27	236
		4	56	252	46	278
	35	0	0	60	0	78
		1	3	103	2	126
		2	14	144	10	165
		3	29	182	23	208
		4	47	218	38	242
	40	0	0	53	0	69
		1	3	91	2	109
		2	12	126	9	146
		3	26	161	20	182
		4	41	194	34	214

NOTE: WHERE CONFIDENCE LEVELS DO NOT FALL ON 90% OR 95%, THE NEXT HIGHEST LEVELS ARE SELECTED.

APPENDIX B GRAPH I

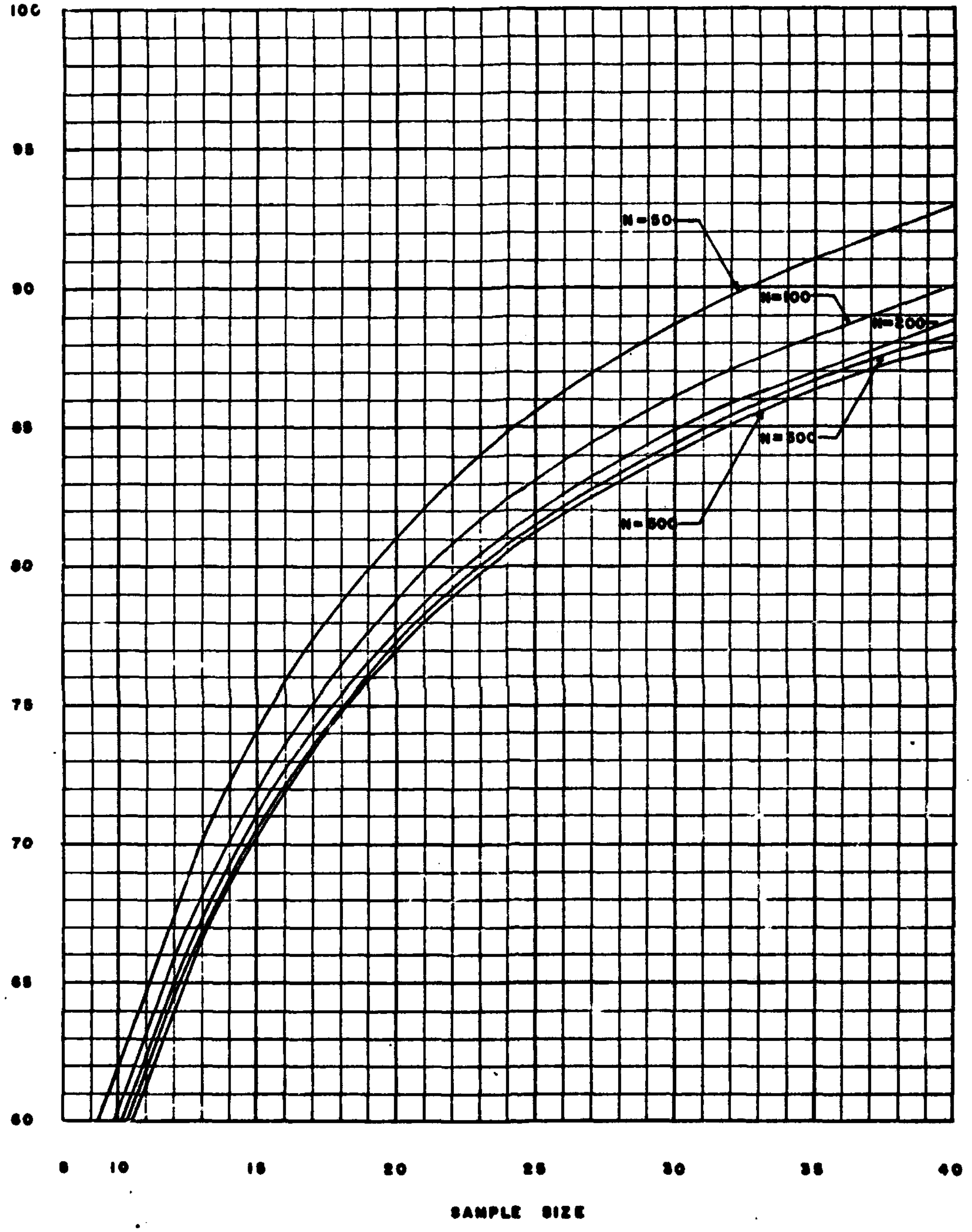


PERCENT OPERABILITY VERSUS FIXED SAMPLE SIZE
.90 CONFIDENCE LEVEL FOR NO DEFECTIVE



PERCENT OPERABILITY VERSUS FIXED SAMPLE SIZE
.90 CONFIDENCE LEVEL FOR ONE DEFECTIVE

APPENDIX B GRAPH 3



PERCENT OPERABILITY VERSUS FIXED SAMPLE SIZE
.90 CONFIDENCE LEVEL FOR TWO DEFECTIVES

APPENDIX C: SAMPLE OF COMPUTER PRINT-OUT OF PROBABILITIES.

```
                                        HYPERGEOMETRIC SERIES

S   N   D   N         P               SUM           CONF              INTERVAL                LEFT SUM        RIGHT SUM

4   50  0   0    0.9999999999    0.9999999999     0.0980
4   50  0   1    0.9199999999    1.9199999999     0.1882    0   0.0000    1   0.0200     0.0000000000    0.9199999999
4   50  0   2    0.8448979591    2.7648979591     0.2710    0   0.0000    2   0.0400     0.0000000000    1.7648979591
4   50  0   3    0.7744897959    3.5393877550     0.3469    0   0.0000    3   0.0600     0.0000000000    2.5393877551
4   50  0   4    0.7085757707    4.2479635258     0.4164    0   0.0000    4   0.0800     0.0000000000    3.2479635258
4   50  0   5    0.6469604863    4.8949240121     0.4790    0   0.0000    5   0.1000     0.0000000000    3.8949240121
4   50  0   6    0.5894528875    5.4843768996     0.5376    0   0.0000    6   0.1200     0.0000000000    4.4843768996
4   50  0   7    0.5358662613    6.0202431610     0.5902    0   0.0000    7   0.1400     0.0000000000    5.0202431610
4   50  0   8    0.4860182370    6.5062613981     0.6378    0   0.0000    8   0.1600     0.0000000000    5.5062613981
4   50  0   9    0.4397307859    6.9459921841     0.6809    0   0.0000    9   0.1800     0.0000000000    5.9459921841

4   50  0   10   0.3968302214   - 3428224055      0.7198    0   0.0000    10  0.2000     0.0000000000    6.3428224055
4   50  0   11   0.3571471993    7.6999696048     0.7548    0   0.0000    11  0.2200     0.0000000000    6.6999696048
4   50  0   12   0.3205167173    8.0204863221     0.7863    0   0.0000    12  0.2400     0.0000000000    7.0204863221
4   50  0   13   0.2867781155    8.3072644376     0.8144    0   0.0000    13  0.2600     0.0000000000    7.3072644376
4   50  0   14   0.2557750759    8.5630395136     0.8395    0   0.0000    14  0.2800     0.0000000000    7.5630395136
4   50  0   15   0.2273556231    8.7903951367     0.8618    0   0.0000    15  0.3000     0.0000000000    7.7903951367
4   50  0   16   0.2013721233    8.9917672600     0.8815    0   0.0000    16  0.3200     0.0000000000    7.9917672600
4   50  0   17   0.1776812852    9.1694485453     0.8989    0   0.0000    17  0.3400     0.0000000000    8.1694485453
4   50  0   18   0.1561441597    9.3255927051     0.9142    0   0.0000    18  0.3600     0.0000000000    8.3255927051
4   50  0   19   0.1366261398    9.4622188449     0.9276    0   0.0000    19  0.3800     0.0000000000    8.4622188449

4   50  0   20   0.1189969604    9.5812158054     0.9393    0   0.0000    20  0.4000     0.0000000000    8.5812158054
4   50  0   21   0.1031306990    9.6843465045     0.9494    0   0.0000    21  0.4200     0.0000000000    8.6843465045
4   50  0   22   0.0889057750    9.7732522796     0.9581    0   0.0000    22  0.4400     0.0000000000    8.7732522796
4   50  0   23   0.0762049500    9.8494572296     0.9656    0   0.0000    23  0.4600     0.0000000000    8.8494572296
4   50  0   24   0.0649153278    9.9143725575     0.9719    0   0.0000    24  0.4800     0.0000000000    8.9143725575
4   50  0   25   0.0549283543    9.9693009118     0.9773    0   0.0000    25  0.5000     0.0000000000    8.9693009118
4   50  0   26   0.0461398176   10.0154407294     0.9819    0   0.0000    26  0.5200     0.0000000000    9.0154407294
4   50  0   27   0.0384498480   10.0538905775     0.9856    0   0.0000    27  0.5400     0.0000000000    9.0538905775
4   50  0   28   0.0317629179   10.0856534954     0.9887    0   0.0000    28  0.5600     0.0000000000    9.0856534954
4   50  0   29   0.0259878419   10.1116413373     0.9913    0   0.0000    29  0.5800     0.0000000000    9.1116413373

4   50  0   30   0.0210377768   10.1326791141     0.9933    0   0.0000    30  0.6000     0.0000000000    9.1326791141
4   50  0   31   0.0168302214   10.1495093356     0.9950    0   0.0000    31  0.6200     0.0000000000    9.1495093356
4   50  0   32   0.0132870169   10.1627963525     0.9963    0   0.0000    32  0.6400     0.0000000000    9.1627963525
4   50  0   33   0.0103343465   10.1731306990     0.9973    0   0.0000    33  0.6600     0.0000000000    9.1731306990
4   50  0   34   0.0079027355   10.1810334346     0.9981    0   0.0000    34  0.6800     0.0000000000    9.1810334346
4   50  0   35   0.0059270516   10.1869604863     0.9987    0   0.0000    35  0.7000     0.0000000000    9.1869604863
4   50  0   36   0.0043465045   10.1913069908     0.9991    0   0.0000    36  0.7200     0.0000000000    9.1913069908
4   50  0   37   0.0031046461   10.1944116369     0.9994    0   0.0000    37  0.7400     0.0000000000    9.1944116369
4   50  0   38   0.0021493703   10.1965610073     0.9996    0   0.0000    38  0.7600     0.0000000000    9.1965610073
4   50  0   39   0.0014329135   10.1979939209     0.9998    0   0.0000    39  0.7800     0.0000000000    9.1979939209

4   50  0   40   0.0009118541   10.1989057750     0.9998    0   0.0000    40  0.8000     0.0000000000    9.1989057750
4   50  0   41   0.0005471124   10.1994528875     0.9999    0   0.0000    41  0.8200     0.0000000000    9.1994528875
4   50  0   42   0.0003039513   10.1997568388     0.9999    0   0.0000    42  0.8400     0.0000000000    9.1997568388
4   50  0   43   0.0001519756   10.1999088145     0.9999    0   0.0000    43  0.8600     0.0000000000    9.1999088145
4   50  0   44   0.0000651324   10.1999739470     0.9999    0   0.0000    44  0.8800     0.0000000000    9.1999739470
4   50  0   45   0.0000217108   10.1999956578     0.9999    0   0.0000    45  0.9000     0.0000000000    9.1999956578
4   50  0   46   0.0000043421   10.1999999999     0.9999    0   0.0000    46  0.9200     0.0000000000    9.1999999999
4   50  1   12   0.4395657837    0.4395657837     0.0430
4   50  1   13   0.4386018237    0.8781676074     0.0860    12  0.2400    13  0.2600     0.0000000000    0.4386018237
4   50  1   11   0.4365132435    1.3146808510     0.1280    11  0.2200    13  0.2600     0.4365132435    0.4386018237

4   50  1   14   0.4340425531    1.7487234042     0.1714    11  0.2200    14  0.2800     0.4365132435    0.8726443768
4   50  1   10   0.4290056448    2.1777290490     0.2135    10  0.2000    14  0.2800     0.8655188884    0.8726443768
4   50  1   15   0.4262917933    2.6040208423     0.2552    10  0.2000    15  0.3000     0.8655188884    1.2989361702
4   50  1   9    0.4165870603    3.0206079027     0.2961    9   0.1800    15  0.3000     1.2821059487    1.2989361702
4   50  1   16   0.4157359965    3.4363438992     0.3368    9   0.1800    16  0.3200     1.2821059487    1.7146721667
4   50  1   17   0.4027442466    3.8390881458     0.3763    9   0.1800    17  0.3400     1.2821059487    2.1174164133
4   50  1   8    0.3987841945    4.2378723404     0.4154    8   0.1600    17  0.3400     1.6808901432    2.1174164133
4   50  1   18   0.3876682587    4.6255405992     0.4534    8   0.1600    18  0.3600     1.6808901432    2.5050846721
4   50  1   7    0.3751063829    5.0006469821     0.4902    7   0.1400    18  0.3600     2.0559965262    2.5050846721
4   50  1   19   0.3708423795    5.3714893616     0.5266    7   0.1400    19  0.3800     2.0559965262    2.8759270516

4   50  1   20   0.3525835866    5.7240729483     0.5611    7   0.1400    20  0.4000     2.0559965262    3.2285106382
4   50  1   6    0.3450455927    6.0691185410     0.5950    6   0.1200    20  0.4000     2.4010421189    3.2285106382
4   50  1   21   0.3331914893    6.4023100303     0.6276    6   0.1200    21  0.4200     2.4010421189    3.5617021276
4   50  1   22   0.3129483282    6.7152583586     0.6583    6   0.1200    22  0.4400     2.4010421189    3.8746504559
4   50  1   5    0.3080764220    7.0233347807     0.6885    5   0.1000    22  0.4400     2.7091185410    3.8746504559
4   50  1   23   0.2921189792    7.3154537599     0.7172    5   0.1000    23  0.4600     2.7091185410    4.1667694351
4   50  1   24   0.2709509335    7.5864046895     0.7437    5   0.1000    24  0.4800     2.7091185410    4.4377203647
4   50  1   4    0.2636561007    7.8500607902     0.7696    4   0.0800    24  0.4800     2.9727746417    4.4377203647
4   50  1   25   0.2496743378    8.0997351280     0.7940    4   0.0800    25  0.5000     2.9727746417    4.6873947025
4   50  1   26   0.2285019539    8.3282370820     0.8164    4   0.0800    26  0.5200     2.9727746417    4.9158966565

4   50  1   3    0.2112244897    8.5394615718     0.8372    3   0.0600    26  0.5200     3.1839991315    4.9158966565
4   50  1   27   0.2076291793    8.7470907511     0.8575    3   0.0600    27  0.5400     3.1839991315    5.1235258358
4   50  1   28   0.1872340425    8.9343247937     0.8759    3   0.0600    28  0.5600     3.1839991315    5.3107598784
4   50  1   29   0.1674772036    9.1018019973     0.8923    3   0.0600    29  0.5800     3.1839991315    5.4782370820
4   50  1   2    0.1502040816    9.2520060790     0.9070    2   0.0400    29  0.5800     3.3342032131    5.4782370820
4   50  1   30   0.1485019537    9.4005080329     0.9216    2   0.0400    30  0.6000     3.3342032131    5.6267390360
4   50  1   31   0.1304342162    9.5309422492     0.9344    2   0.0400    31  0.6200     3.3342032131    5.7571732522
4   50  1   32   0.1133825445    9.6443247937     0.9455    2   0.0400    32  0.6400     3.3342032131    5.8705557967
4   50  1   33   0.0974301241    9.7417620179     0.9550    2   0.0400    33  0.6600     3.3342032131    5.9679859208
4   50  1   34   0.0826747720    9.8244376899     0.9631    2   0.0400    34  0.6800     3.3342032131    6.0506606930

4   50  1   1    0.0799999999    9.9044376899     0.9710    1   0.0200    34  0.6800     3.4142032131    6.0506606930
4   50  1   35   0.0691489361    9.9735866261     0.9778    1   0.0200    35  0.7000     3.4142032131    6.1198096291
4   50  1   36   0.0568996960   10.0304863221     0.9833    1   0.0200    36  0.7200     3.4142032131    6.1767093252
4   50  1   37   0.0459407624   10.0764270846     0.9878    1   0.0200    37  0.7400     3.4142032131    6.2226500877
4   50  1   38   0.0363004776   10.1127395622     0.9914    1   0.0200    38  0.7600     3.4142032131    6.2589505653
4   50  1   39   0.0279418150   10.1406773773     0.9941    1   0.0200    39  0.7800     3.4142032131    6.2868923803
4   50  1   40   0.0208423795   10.1615197568     0.9962    1   0.0200    40  0.8000     3.4142032131    6.3077347598
4   50  1   41   0.0149544072   10.1764741641     0.9976    1   0.0200    41  0.8200     3.4142032131    6.3226891671
4   50  1   42   0.0102127659   10.1866869300     0.9986    1   0.0200    42  0.8400     3.4142032131    6.3329019331
4   50  1   43   0.0065347544   10.1932218844     0.9993    1   0.0200    43  0.8600     3.4142032131    6.3394368875
```

APPENDIX D: SYMBOLIC LANGUAGE PROGRAM FOR IBM 1401.

```
PG LIN  CT  LABEL   OP   A OPERAND      B OPERAND      D
 1 010              CTL  441
 1 020                  * HYPERGEOMETRIC SERIES
 1 030                  * SUM FROM BOTH SIDES OF MAX LIKELIHOOD
 1 040                  * SENSE SWITCH G 75 TO 99 PERCENT CONF.
 1 050  32  TABLE   DCW  *          HYPERGEOMETRIC SERIES
[illegible]
```

```
PG LIN  CT  LABEL   OP   A OPERAND      B OPERAND      D
[illegible]
```

D-1

APPENDIX D: CONTINUED.

```
PG LIN  CT  LABEL     OP    A OPERAND          B OPERAND          D

12 040  7             MZ    DIV      +009      DIV      +010
12 050  7             ZA    MS                 MULT     .004
12 060  7             A     BCU1               MULT     +004
12 070  7             B     MULT     +004      DIV      +006
12 080  7             MCW   DIV      +019      RSUM
12 090  7             ZA    CON00              MULT     +001
12 100  7             B     RSUM               MULT     +022
12 110  7             MCW   MULT     +020      LEV00
12 120  7             ZA    CON05              MULT     +001
12 130  7             B     RSUM               MULT     +022
12 140  7             MCW   MULT     +020      LEV05
12 150  7             ZA    CON90              MULT     +001
12 160  7             B     RSUM               [illegible]
12 170  7             MCW   MULT     +020      LEV90
12 180  7             ZA    CON95              MULT     +001
12 190  7             B     RSUM               MULT     +022
13 010  7             MCW   MULT     +020      LEV95
13 020  7             ZA    CON99              MULT     +001
13 030  7             B     RSUM               MULT     +022
13 040  7             MCW   MULT     +020      LEV99
13 050  7             [illegible] CONEX1       MULT     +002
13 060  7             [illegible] RSUM         MULT     +023
13 070  7             [illegible] MULT +020    LEVEX1
13 080  7             [illegible] [illegible]X2 MULT    +002
13 090  7             [illegible] RSUM         MULT     +023
13 100  7             [illegible] MULT +020    LEVEX2
13 110  4             B     [illegible]
14 010  7   [illegible] ZA  NMAT               A
14 020  7             S     MS                 A
14 030  7             ZA    RS                 B
14 040  7             ZA    MD16               C
14 050  7             A     MS                 C
14 060  7             S     NMAT               C
14 070  7             S     MS                 C
14 080  7             ZA    MS                 D
14 090  7             S     ML                 D
14 100  8             BWZ   AREAD1             A                  K
14 110  8             BWZ   AREAD1             B                  K
14 120  8             BWZ   AREAD1             C                  K
14 130  8             BWZ   AREAD1             D                  K
14 140  7             ZA    A                  NP
14 150  7             A     B                  NP
14 160  7             A     C                  NP
14 170  7             A     D                  NP
14 180  7             MZ    NP       -001      NP
14 190  7             ZA    A                  MARG1
14 200  7             A     C                  MARG1
15 010  7             ZA    C                  MARG2
15 020  7             A     D                  MARG2
15 030  7             ZA    A                  MARG3
15 040  7             A     B                  MARG3
15 050  7             ZA    B                  MARG4
15 060  7             A     D                  MARG4
15 070  7             MZ    MARG1    -001      MARG1
15 080  7             MZ    MARG2    -001      MARG2
15 090  7             MZ    MARG3    -001      MARG3
15 100  7             MZ    MARG4    -001      MARG4
15 110  7             ZA    A                  MULT     +003
15 120  7             S     BCU1               MULT     +003
15 130  8             BWZ   CKB                MULT     +003      B
15 140  7             MCW   [illegible]        A
15 150  7   CKB       ZA    B                  MULT     +003
15 160  7             S     BCU1               MULT     +003
15 170  8             BWZ   CKC                MULT     +003      B
15 180  7             MCW   [illegible]        B
15 190  7   CKC       ZA    C                  MULT     +003
15 200  7             S     BCU1               MULT     +003
16 010  8             BWZ   CKD                MULT     +003      B
16 020  7             MCW   [illegible]        C
16 030  7   CKD       ZA    D                  MULT     +003
16 040  7             S     BCU1               MULT     +003
16 050  8             BWZ   [illegible]        MULT     +003      B
16 060  7             MCW   [illegible]        D
16 070  7   [illegible] ZA  [illegible]        0106
16 080  7             MCW   [illegible]        0094
16 090  7   [illegible] MCW 0094               0099
16 100  7   ADST1     A     [illegible]        0099
16 110  7             C     [illegible]   1    [illegible]   2
16 120  5             B     [illegible]                           T
16 130  4             B     COMP16
16 140  7   [illegible] MCW [illegible]   2    [illegible]
16 150  7             MCW   [illegible]   1    [illegible]   2
16 160  7             MCW   [illegible]        [illegible]   1
16 170  7   COMP16    C     [illegible]        [illegible]
16 180  5             B     [illegible]                           V
16 190  7             A     [illegible]        0094
16 200  7             C     0094               [illegible]
17 010  5             B     [illegible]                           V
17 020  7             ZA    [illegible]        TESTZE
17 030  7             S     [illegible]        TESTZE
17 040  8             BWZ   [illegible]        TESTZE             B
17 050  7             ZA    [illegible]        [illegible]
17 060  7   [illegible] ZA  [illegible]        TESTZE
17 070  7             S     [illegible]        TESTZE
17 080  8             BWZ   [illegible]        TESTZE             B
17 090  7             ZA    [illegible]        [illegible]
17 100  7   [illegible] ZA  [illegible]        TESTZE
17 110  7             S     [illegible]        TESTZE
17 120  8             BWZ   [illegible]        TESTZE             B
17 130  7             ZA    [illegible]        [illegible]
17 140  7   [illegible] ZA  [illegible]        TESTZE
17 150  7             S     [illegible]        TESTZE
17 160  8             BWZ   [illegible]        TESTZE             B
17 170  7             ZA    [illegible]        [illegible]
17 180  7   [illegible] [illegible] [illegible] [illegible]
17 190  7             ZA    [illegible]        DIV      +020
17 200  7             MCW   [illegible]        DIV      +010
[illegible] 7         MCW   [illegible]        [illegible]
[illegible] 7 [illegible] C NP                 [illegible]
[illegible] 5         B     [illegible]                           B
[illegible] 7         B     NP                 DIV      +006
[illegible] 7         ZA    DIV      +019      MULT     +010
[illegible] 8 [illegible] BWZ [illegible]      [illegible]        1
[illegible] 7         ZA    MULT     +019      TESTZE
[illegible] 7         S     [illegible]        TESTZE
[illegible] 8         BWZ   [illegible]        TESTZE             K
[illegible] 7 [illegible] ZA MULT    +019      DIV      +025
[illegible] 7         S     [illegible]        NP
[illegible] 7         MZ    NP       -001      NP
```

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PG LIN  CT  LABEL     OP    A OPERAND          B OPERAND          D

18 130  4             B     [illegible]
18 140  7   [illegible] ZA  A                  NP
18 150  7             MZ    NP       -001      NP
18 160  7             ZA    NP                 TESTZE
18 170  7             S     BCU1               TESTZE
18 180  8             BWZ   MULTNU             TESTZE             K
18 190  7             ZA    B                  A
18 200  7             ZA    C                  B
18 210  7             ZA    D                  C
18 220  7             ZA    ZERO               D
19 010  7             ZA    BCU1               MARG1
19 020  7             MZ    MARG1    -001      MARG1
19 030  4             B     [illegible]
19 040  7   MULTPR    ZA    MARGT              TESTZE
19 050  7             S     BCU2               TESTZE
19 060  8             BWZ   [illegible]        TESTZE             K
19 070  7             M     MARGT              MULT     +029
19 080  7             MCW   MULT     +025      MDCU
19 090  7             ZA    MDCU               MULT     +019
19 100  7             S     BCU1               MARGT
19 110  8             BWZ   MULTPR             [illegible]        1
19 120  4             B     [illegible]
19 130  8   MULTNU    BWZ   FINISH             [illegible]        1
19 140  4             [illegible] [illegible]
19 150  4             B     MULTPR
19 160  7   [illegible] ZA  MARG3              MARGT
19 170  7             ZA    MARG4              MARG3
19 180  7             ZA    ZERO               MARG4
19 190  7             ZA    ZERO               TESTZE
19 200  7             S     MARGT              TESTZE
20 010  8             BWZ   MULTPR             TESTZE             K
20 020  8             BWZ   FINISH             [illegible]        1
20 030  4             SW    [illegible]
20 040  4             B     [illegible]
20 050  7   FINISH    CW    [illegible]        [illegible]
20 060  7             MCW   MULT     +019      [illegible]
20 070  7             MCW   MULT     +019      [illegible]
20 080  7             MCW   MULT     +019      [illegible]
20 090  7             ZA    MULT     +019      SUMP21
20 100  7             ZA    ZERO               SUMPR
20 110  7             ZA    ZERO               SUMPL
20 120  7             LCA   LEDT21-005         0237
20 130  7             MCE   SUMP21-005         0237
20 140  7             LCA   LEDT21-005         0259
20 150  7             MCE   SUMP21-005         0259
20 160  7             MCS   MD16               0209
20 170  7             MCS   MS                 0204
20 180  7             LCA   PCT      -005      0219
20 190  4             SW    0101
21 010  7             MCE   NMAT               0219
21 011  7             LCA   PCT      -005      0214
21 012  7             MCS   MS                 0214
21 020  5             B     [illegible]                           B
21 021  7   CONF      ZA    ZERO               DIV      +027
21 022  7             MCW   SUMP21-009         DIV      +023
21 023  7             MZ    DIV      +022      DIV      +023
21 024  7             B     RSUM     -009      DIV      +012
21 025  4             SW    DIV      +008
21 026  7             LCA   PCT                0264
21 027  7             MCE   DIV      +015      0264
21 028  4             CW    DIV      +008
21 032  4             B     [illegible]
21 040  1   [illegible] B
21 050  4             B     CTLINE
21 060  6   PCNCH     NOP   [illegible]                           C
21 090  7             SW    [illegible]        PLEFT
21 100  7             ZA    MD16               [illegible]
21 110  7             S     MS                 [illegible]
21 120  7             ZA    MD16               [illegible]
21 130  7             S     MD16               [illegible]
21 140  7             A     BCU1               [illegible]
21 150  7             ZA    MD16               [illegible]
21 160  7             ZA    MD16               [illegible]
21 170  7             S     MD16               [illegible]
21 180  7             S     MS                 [illegible]
21 200  7             A     MS                 [illegible]
22 010  7             A     BCU1               [illegible]
22 020  7             ZA    MD16               [illegible]
22 023  7             MZ    DIV      +022      DIV      +023
22 030  7             A     BCU1               [illegible]
22 040  7             ZA    MD16               [illegible]
22 050  7             S     MD16               [illegible]
22 060  7             A     RS                 [illegible]
22 070  7             S     RS                 [illegible]
22 080  7             ZA    MD16               [illegible]
22 090  7             S     MS                 [illegible]
22 100  7             A     BCU1               [illegible]
22 110  7             ZA    MD16               [illegible]
22 120  7             S     MD16               [illegible]
22 130  4             B     [illegible]
22 140  6   [illegible] CW  PLEFT
22 150  8             BWZ   [illegible]        LASTR              1
22 160  8   [illegible] BWZ LEFT               [illegible]        1
22 170  7             ZA    [illegible]        TESTZE
22 180  7             S     [illegible]        TESTZE
22 190  8             BWZ   [illegible]        TESTZE             K
22 200  4             B     [illegible]
[illegible] 8 [illegible] BWZ [illegible]      LASTR              1
[illegible] 4         B     [illegible]
[illegible] 8 [illegible] BWZ [illegible]      [illegible]        1
[illegible] 4         B     [illegible]
[illegible] 7 [illegible] SW [illegible]       LASTR
[illegible] 4         B     [illegible]
[illegible] 7 [illegible] ZA [illegible]       TESTZE
[illegible] 7         S     [illegible]        TESTZE
[illegible] 8         BWZ   [illegible]        TESTZE             K
[illegible] 4         B     [illegible]
[illegible] 7 [illegible] SW [illegible]       LASTR
[illegible] 4         B     [illegible]
[illegible] 7 [illegible] ZA [illegible]       MULT     [illegible]
[illegible] 7         B     [illegible]        MULT     [illegible]
[illegible] 7         ZA    [illegible]        DIV      [illegible]
[illegible] 7         MCW   MULT     +010      DIV      [illegible]
[illegible] 7         MZ    DIV      +019      DIV      [illegible]
[illegible] 7         ZA    [illegible]        MULT     [illegible]
[illegible] 7         B     [illegible]        MULT     [illegible]
[illegible] 4         SW    MULT     [illegible]
[illegible] 7         B     MULT     [illegible] DIV    [illegible]
[illegible] 4         CW    MULT     [illegible]
```

APPENDIX D CONTINUED.

[illegible]

U. S. NAVAL AMMUNITION DEPOT
OAHU, HAWAII

1804:sm
5220/NT
3 Jan 1963

From: Commanding Officer
To: Chief, Bureau of Naval Weapons (FQ-1)
Department of the Navy
Washington 25, D.C.

Subj: Tables of the Hypergeometric Distribution Functions

Encl: (1) Mathematical derivation of the tables
(2) Tables of point and cumulative probabilities for various sample and population size combinations

1. Enclosures (1) and (2) are forwarded for publication by the U. S. Government Printing Office. Enclosure (1) contains mathematical derivation of the hypergeometric distribution function, tables, graphs and IBM 1401 computer program. Enclosure (2) (forwarded under separate cover) contains two copies of the tabulations of the hypergeometric probabilities.

2. The tables, which include point and cumulative probabilities are designed primarily for use by personnel familiar with statistics to estimate stockpile quality level.

3. Copy addressees are advised that copies of the tabular presentation, enclosure (2), which are quite voluminous cannot be made available by this command. It is presumed that they will be generally available through the Bureau of Naval Weapons or the Government Printing Office if the Bureau of Naval Weapons decides to have them published.

H. H. MEEKER, JR.
By direction

Copy to:
Distribution list (w/o encl (2))